Workshop on Lie algebroids and Lie groupoids in differential geometry

Sheffield, October 2007

Abstracts

Ulrich Bunke, *Regensburg*

Duality of topological abelian group stacks and applications

I will first motivate the notion of a topological abelian group stack by examples. The main objective of the talk is to explain how Pontrjagin duality of locally compact groups extends to the stacky world. I will also discuss some basic calculations of the cohomology of sheaves generated by topological groups which enter into the discussion of duality.

Alberto Cattaneo, *Zürich*

Reduction of Courant algebroids and generalized complex structures from the viewpoint of graded geometry

After Roytenberg, Courant algebroids may be regarded as graded symplectic manifolds endowed with a Poisson self–commuting function. A generalized complex manifold may be regarded as such an object endowed with a second function satisfying one condition, also expressible in terms of Poisson brackets. As a consequence, reduction of Courant algebroids and generalized complex structures may be regarded as the usual symplectic reduction (now in the graded sense). In this talk (based on work in progress with Bursztyn, Mehta and Zambon) I will explain how to reformulate this in terms of classical (i.e., nongraded) differential geometry, recovering and extending the existing reduction procedures.

Andre Henriques, *Münster*

String bundles with connections

Given a String bundle, there are many possible notions of what one might mean by a connection. I was unhappy with all of them. Last month, I talked with Jacob Lurie about that problem, and he told me of another possible notion that indeed looks better. I don't know if I'll have really understood it by the time I give my talk, but I'll try ...

Yvette Kosmann-Schwarzbach, *Ecole Polytechnique*

Variations on Poisson-Nijenhuis structures

We first explained how to study Poisson-Nijenhuis structures on a Lie algebroid A in terms of the big bracket, the canonical even graded bracket on the cotangent bundle of the supermanifold ΠA obtained by changing the parity of the fibres of the vector bundle A. The vanishing of the Nijenhuis torsion of a $(1,1)$-tensor is a sufficient condition for the deformed bracket $[\ ,\ ]_N$ to be a Lie bracket, compatible with the initial bracket, just as the vanishing of the Schouten bracket of a bivector π is a sufficient condition for the Fuchssteiner-Magri-Morosi bracket $[\ ,\ ]_\pi$ to be a Lie bracket, such that (A, A^*) is a Lie bialgebroid. The compatibility condition that characterizes the Poisson-Nijenhuis structures is a sufficient condition for the pair $([\ ,\ ]_N, [\ ,\ ]_\pi)$ to define a Lie bialgebroid structure. If the Lie algebroid A is the tangent bundle of a manifold, this condition is also necessary.

The defining axioms of a Poisson-quasi-Nijenhuis manifold in the sense of Stiénon and Xu can be generalized to the case of a Lie algebroid with a background closed 3-form ψ. We proved that they are sufficient conditions for $[\ ,\ ]_N$, $[\ ,\ ]_\pi$ and ψ to define a quasi-Lie bialgebroid structure. Stiénon and Xu proved that each generalized complex structure on the standard Courant algebroid $TM \oplus T^*M$ defines a Poisson-quasi-Nijenhuis structure, the 3-form of which is exact. We showed that this result extends to a Courant algebroid which is the double of a Lie bialgebroid.

The axioms of the twisted Poisson-quasi-Nijenhuis structures, introduced by Zucchini as necessary and sufficient conditions for the action of the Hitchin sigma-model to satisfy the Batalin-Vilkovisky master equation, can similarly be extended to an arbitrary Lie algebroid. We conjecture that they are sufficient conditions for the existence of a quasi-Lie bialgebroid structure involving both background 3-forms entering into this sigma-model.

Urs Schreiber, *Hamburg*

String and Chern–Simons Lie n–algebras

For every transgressive element in Lie algebra cohomology, there is a short exact sequence of Lie n–algebras. Higher connections taking values in these yield the Chern–Simons functional and Chern classes. Applications concern connections on String bundles and Chern–Simons theory.

Ted Voronov, *Manchester*

Q–manifolds and Mackenzie theory

Q–manifolds are supermanifolds endowed with a homological vector field, i.e., a self–commuting odd vector field. They should be regarded as a non–linear extension of the notion of Lie algebras, together with Poisson and Schouten manifolds. Q–manifolds provide an effective geometric language for describing structures such as, e.g., strong homotopy Lie algebras and Lie algebroids.

"Mackenzie theory" stands for the rich circle of notions that have been put forward by Kirill Mackenzie in recent years (solo or in collaboration): double structures such as double Lie groupoids and double Lie algebroids, Lie bialgebroids and their doubles, nontrivial dualities for double and multiple vector bundles, etc.

In the talk I shall discuss double Lie algebroids (discovered by Mackenzie) and explain how this quite complicated fundamental notion becomes equivalent to a very simple one if the language of Q–manifolds is used. In particular, it shows how the two seemingly different notions of a "Drinfeld double" of a Lie bialgebroid due to Mackenzie and Roytenberg respectively, turn out to be the same object if properly understood. It also allows to obtain generalizations such as multiple Lie algebroids and multiple Lie bialgebroids.

30 minute talks

Iakovos Androulidakis, *Crete*

The holonomy groupoid of a singular foliation

We present the construction of the holonomy groupoid associated with a general (Stefan-Sussmann) foliation and discuss briefly the first implications of our construction to Noncommutative Geometry. Namely, how our method allows the construction of the C^*–algebra of the foliation, the definition of pseudodifferential calculus along the leaves, as well as the definition of the analytic index map for such operators. This is joint research with George Skandalis, part of which is work in progress.

David Martinez, *Utrecht*

Poisson manifolds of compact type and integral affine structures